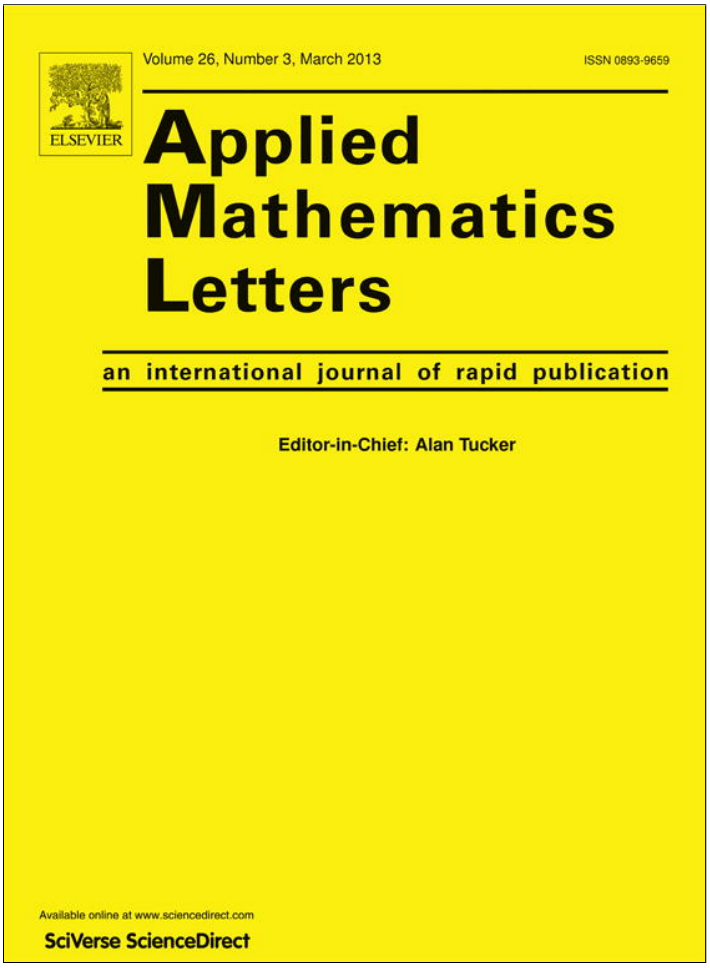
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Analytical bending solutions of clamped rectangular thin plates resting on elastic foundations by the symplectic superposition method

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ABSTRACT

In this paper, the analytical bending solutions of clamped rectangular thin plates resting on elastic foundations are obtained by a rational symplectic superposition method which is based on the Hamiltonian system. The proposed method is capable of solving the plate problems with different boundary conditions via a step-by-step derivation without any trial solutions. The presented solution procedure can be extended to more boundary value problems in engineering.

1. Introduction

The bending problems of rectangular plates exist in a vast variety of engineering structures such as rigid pavements, water gates and bridge decks. In view of the importance of such problems, researchers have been investigating the solutions by various methods, either analytical [1–7] or numerical [8–16]. Until now, most of the validated methods are numerical or approximate, which cannot satisfy exactly the governing partial differential equation as well as the boundary conditions of plates. For the analytical methods, one can hardly find one to seek the solutions rationally because the current methods are basically semi-inverse or based on the semi-inverse methods, in which the preset functions (for example, deflections or stress functions) are necessary. This shortcoming limits the scope of the application of the semi-inverse methods.

A rational symplectic superposition method [17,18] is further developed to deal with an important class of problems in engineering, the bending of rectangular thin plates resting on elastic foundations, of which we focus on the clamped plates in this work. The Hamiltonian system based dual equations are constructed, by which the problems are transferred into the symplectic space in comparison with the Lagrange system in Euclid space. Then the separation of variables and expansion of symplectic eigen vector are implemented, respectively, to yield the analytical solutions of fundamental systems, i.e., the simply supported plates on elastic foundations, which are regarded as the first class of sub-problems. The solutions of other two classes of sub-problems are derived accordingly. The superposition of the three sub-problems is equivalent to the original problem, and gives the solution for a clamped plate resting on an elastic foundation. Compared with the classical symplectic method [3–5], the complex transcendent eigenvalue equations are avoided, which cannot be solved analytically. Besides, the boundary conditions are satisfied via systems of simple linear equations rather than by the variational equations in the complex field. These advantages ensure the general applicability of the present method.

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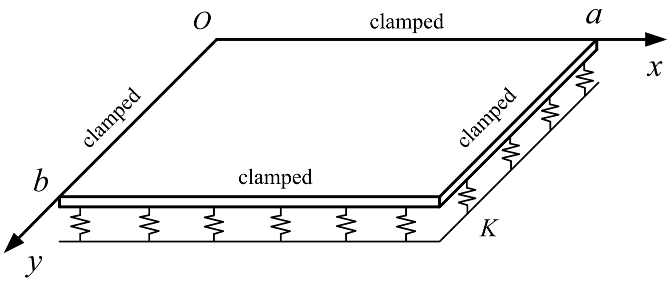

Fig. 1. A clamped rectangular thin plate resting on an elastic foundation.

The present work is the first attempt to obtain the analytical solutions of foundation plates using the symplectic superposition method. The finite element analysis by the ABAQUS package is provided to verify the derivation, by which the accuracy of the obtained results is confirmed.

2. Hamiltonian dual equations for elastic foundation plates

Fig. 1 illustrates the coordinate system of a clamped rectangular thin plate resting on an elastic Winkler foundation, with the dimensions a and b in the x and y direction.

The equilibrium equations of the plate are

$$\partial M_x/\partial x + \partial M_{xy}/\partial y - Q_x = 0 \tag{1}$$

$$\partial M_y/\partial y + \partial M_{xy}/\partial x - Q_y = 0 \tag{2}$$

$$\partial Q_x/\partial x + \partial Q_y/\partial y + q - Kw = 0 \tag{3}$$

while the internal forces are

$$\begin{aligned} &M_x = -D(\partial^2 w/\partial x^2 + \nu \partial^2 w/\partial y^2) && \text{(a)} \\ &M_y = -D(\partial^2 w/\partial y^2 + \nu \partial^2 w/\partial x^2) && \text{(b)} \\ &M_{xy} = -D(1-\nu)\partial^2 w/\partial x \partial y && \text{(c)} \end{aligned} \tag{4}$$

$$Q_x = -D\partial(\nabla^2 w)/\partial x; \qquad Q_y = -D\partial(\nabla^2 w)/\partial y \quad \text{(a), (b)} \tag{5}$$

$$V_x = Q_x + \partial M_{xy}/\partial y; \qquad V_y = Q_y + \partial M_{xy}/\partial x \quad \text{(a), (b)} \tag{6}$$

where K denotes the foundation modulus, w the transverse displacement of the plate midplane, D the flexural rigidity, ν the Poisson's ratio, q the distributed transverse load, M_x and M_y are the bending moments, M_{xy} is the torsional moment, Q_x and Q_y are the shear forces, V_x and V_y are the equivalent shear forces, respectively.

Eqs. (3) and (6)(a), (b) yield

$$\partial V_x/\partial x + \partial V_y/\partial y - 2\partial M_{xy}/\partial x \partial y + q - Kw = 0. \tag{7}$$

By introducing

$$\partial w/\partial y = \theta \tag{8}$$

and Eq. (4)(b), we have

$$\partial \theta/\partial y = -\nu \partial^2 w/\partial x^2 - M_y/D. \tag{9}$$

From Eq. (4)(c),

$$M_{xy} = -D(1-\nu)\partial \theta/\partial x. \tag{10}$$

Eqs. (4)(b), (c), (5)(a), (6)(a) and (7) lead to

$$\partial V_y/\partial y = D(1-\nu^2)\partial^4 w/\partial x^4 - \nu \partial^2 M_y/\partial x^2 - q + Kw. \tag{11}$$

From Eqs. (2), (6)(b) and (10), we obtain

$$\partial M_y/\partial y = V_y + 2D(1-\nu)\partial^2 \theta/\partial x^2. \tag{12}$$

Introducing $V_y = -T$, Eq. (8), (9), (11) and (12) are integrated in the matrix form

$$\partial \mathbf{Z}/\partial y = \mathbf{HZ} + \mathbf{f} \tag{13}$$

where $\mathbf{H} = \begin{bmatrix} \mathbf{F} & \mathbf{G} \\ \mathbf{Q} & -\mathbf{F}^{\mathrm{T}} \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} D(\nu^2-1)\partial^4/\partial x^4 - K & 0 \\ 0 & 2D(1-\nu)\partial^2/\partial x^2 \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -\nu\partial^2/\partial x^2 & 0 \end{bmatrix}$, and $\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & -1/D \end{bmatrix}$. $\mathbf{Z} = \left[w, \theta, T, M_y\right]^{\mathrm{T}}$ is the state vector of the plate. $\mathbf{f} = [0, 0, q, 0]^{\mathrm{T}}$ is the vector with respect to the external load q. Observing

$\mathbf{H}^{\mathrm{T}} = \mathbf{JHJ}$, where $\mathbf{J} = \begin{bmatrix} 0 & \mathbf{I}_2 \\ -\mathbf{I}_2 & 0 \end{bmatrix}$ is the symplectic matrix in which $\mathbf{I}_2$ is the 2×2 unit matrix, $\mathbf{H}$ is a Hamiltonian operator matrix [4]. Via the above derivation, we obtain the Hamiltonian dual equations for a rectangular thin plate on a Winkler foundation in the form of Eq. (13).

3. Analytical bending solutions of clamped rectangular thin plate resting on elastic foundations

Without loss of generality, the bending problem of a clamped rectangular thin plate resting on a Winkler foundation subjected to a concentrated load P at (x_0, y_0) is considered. The problem is divided into three sub-problems: (1) the bending of a simply supported rectangular thin plate resting on the Winkler foundation subjected to the concentrated load P at (x_0, y_0); (2) the bending of the plate with the moments represented by $\sum_{n=1}^{\infty} E_n \sin(n\pi x/a)$ and $\sum_{n=1}^{\infty} F_n \sin(n\pi x/a)$ distributed along the edges $y = 0$ and $y = b$, respectively; (3) the bending of the plate with the moments represented by $\sum_{n=1}^{\infty} G_n \sin(n\pi y/b)$ and $\sum_{n=1}^{\infty} H_n \sin(n\pi y/b)$ distributed along the edges $x = 0$ and $x = a$, respectively. By superposing the solutions of the three sub-problems, the original problem is solved.

3.1. Analytical solutions of the sub-problems

For the homogeneous equation of Eq. (13)

$$\partial \mathbf{Z}/\partial y = \mathbf{HZ} \tag{14}$$

the separation of variables in the symplectic space is valid, which yields

$$\mathbf{Z} = \mathbf{X}(x)Y(y) \tag{15}$$

where $\mathbf{X}(x) = [w(x), \theta(x), T(x), M_y(x)]^{\mathrm{T}}$ and $Y(y)$ is a function of y. Substituting Eq. (15) into Eq. (14), we have

$$dY(y)/dy = \mu Y(y); \qquad \mathbf{HX}(x) = \mu \mathbf{X}(x) \qquad \text{(a), (b)} \tag{16}$$

where μ is the eigenvalue and $\mathbf{X}(x)$ the corresponding eigenvector. The solution of the Hamiltonian dual equations is thus reduced to an eigenvalue problem.

The eigenvalues and corresponding eigenvectors satisfying the boundary conditions

$$w(x)|_{x=0,a} = M_x(x)|_{x=0,a} = 0 \tag{17}$$

of a plate simply supported at the two opposite edges $x = 0$ and $x = a$ are

$$\begin{aligned} &\mu_{n1} = \sqrt{\alpha_n^2 + R}; \qquad \mu_{n2} = -\sqrt{\alpha_n^2 + R}; \qquad \mu_{n3} = \sqrt{\alpha_n^2 - R}; \qquad \text{(a), (b), (c)} \\ &\mu_{n4} = -\sqrt{\alpha_n^2 - R} \quad (n = 1, 2, 3, \ldots) \qquad \text{(d)} \end{aligned} \tag{18}$$

and

$$\begin{aligned} &\mathbf{X}_{n1}(x) = \{1, \mu_{n1}, D\mu_{n1}[\alpha_n^2(\nu - 1) + R], D[\alpha_n^2(\nu - 1) - R]\}^{\mathrm{T}} \sin(\alpha_n x) \qquad \text{(a)} \\ &\mathbf{X}_{n2}(x) = \{1, -\mu_{n1}, -D\mu_{n1}[\alpha_n^2(\nu - 1) + R], D[\alpha_n^2(\nu - 1) - R]\}^{\mathrm{T}} \sin(\alpha_n x) \qquad \text{(b)} \\ &\mathbf{X}_{n3}(x) = \{1, \mu_{n3}, D\mu_{n3}[\alpha_n^2(\nu - 1) - R], D[\alpha_n^2(\nu - 1) + R]\}^{\mathrm{T}} \sin(\alpha_n x) \qquad \text{(c)} \\ &\mathbf{X}_{n4}(x) = \{1, -\mu_{n3}, -D\mu_{n3}[\alpha_n^2(\nu - 1) - R], D[\alpha_n^2(\nu - 1) + R]\}^{\mathrm{T}} \sin(\alpha_n x) \qquad \text{(d)} \end{aligned} \tag{19}$$

where $\alpha_n = n\pi/a$, $R = j\sqrt{K/D}$, j is the imaginary unit. The above eigenvectors satisfy the symplectic conjugacy or orthogonality, i.e., $\int_0^a \mathbf{X}_{n1}(x)^{\mathrm{T}}\mathbf{J}\mathbf{X}_{n2}(x)dx \neq 0$ and $\int_0^a \mathbf{X}_{n3}(x)^{\mathrm{T}}\mathbf{J}\mathbf{X}_{n4}(x)dx \neq 0$ while all the other combinations of the eigenvectors are orthogonal, for example, $\int_0^a \mathbf{X}_{n1}(x)^{\mathrm{T}}\mathbf{J}\mathbf{X}_{n3}(x)dx = 0$.

The solution of Eq. (13) can be given by expansion of symplectic eigen vector as

$$\mathbf{Z} = \mathbf{X}(x)\mathbf{Y}(y) \tag{20}$$

where

$$\mathbf{X}(x) = [\ldots, \mathbf{X}_{n1}(x), \mathbf{X}_{n2}(x), \mathbf{X}_{n3}(x), \mathbf{X}_{n4}(x), \ldots] \tag{21}$$

$$\mathbf{Y}(y) = [\ldots, Y_{n1}(y), Y_{n2}(y), Y_{n3}(y), Y_{n4}(y), \ldots]^{\mathrm{T}}. \tag{22}$$

Substituting Eq. (20) into Eq. (13), observing

$$\mathbf{HX} = \mathbf{XM} \tag{23}$$

$$\mathbf{f} = \mathbf{XG} \tag{24}$$

where $\mathbf{M} = \mathrm{diag}(\ldots, \mu_{n1}, \mu_{n2}, \mu_{n3}, \mu_{n4}, \ldots)$, $\mathbf{G} = [\ldots, g_{n1}, g_{n2}, g_{n3}, g_{n4}, \ldots]^{\mathrm{T}}$ representing the expansion coefficients of $\mathbf{f}$, we obtain

$$d\mathbf{Y}/dy - \mathbf{MY} = \mathbf{G}. \tag{25}$$

For the first sub-problem, i.e., a simply supported rectangular thin plate resting on the Winkler foundation subjected to the concentrated load P at (x_0, y_0), we get from Eq. (24) the expansion coefficients

$$\begin{aligned} &g_{n1} = P\delta(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n1}); \qquad g_{n2} = -P\delta(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n1}) \quad \text{(a), (b)} \\ &g_{n3} = -P\delta(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n3}); \qquad g_{n4} = P\delta(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n3}) \quad \text{(c), (d)} \end{aligned} \tag{26}$$

where $\delta(y-y_0)$ is the Dirac delta function. Substitution of Eqs. (26)(a)–(d) into Eq. (25) yields

$$\begin{aligned} Y_{n1} &= c_{n1}e^{\mu_{n1}y} + Pe^{\mu_{n1}(y-y_0)}H(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n1}) && \text{(a)} \\ Y_{n2} &= c_{n2}e^{-\mu_{n1}y} - Pe^{-\mu_{n1}(y-y_0)}H(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n1}) && \text{(b)} \\ Y_{n3} &= c_{n3}e^{\mu_{n3}y} - Pe^{\mu_{n3}(y-y_0)}H(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n3}) && \text{(c)} \\ Y_{n4} &= c_{n4}e^{-\mu_{n3}y} + Pe^{-\mu_{n3}(y-y_0)}H(y-y_0)\sin(\alpha_n x_0)/(2aDR\mu_{n3}) && \text{(d)} \end{aligned} \tag{27}$$

where $H(y-y_0)$ is the Heaviside theta function, $c_{n1} - c_{n4}$ are the constants to be determined by imposing the boundary conditions at $y = 0$ and $y = b$

$$w|_{y=0,b} = M_y|_{y=0,b} = 0. \tag{28}$$

Substituting Eqs. (19)(a)–(d) and (27)(a)–(d) into Eq. (20) then into Eq. (28), the constants are obtained, by which we obtain the analytical solution of the first sub-problem

$$\begin{aligned} w_1(x,y) = \frac{P}{aDR}\sum_{n=1}^{\infty}&\sin(\alpha_n x_0)\sin(\alpha_n x) \times \{\operatorname{csch}(b\mu_{n3})\sinh(\mu_{n3}y)\sinh[\mu_{n3}(b-y_0)]/\mu_{n3} \\ &- \operatorname{csch}(b\mu_{n1})\sinh(\mu_{n1}y)\sinh[\mu_{n1}(b-y_0)]/\mu_{n1} \\ &+ [\sinh[\mu_{n1}(y-y_0)]/\mu_{n1} - \sinh[\mu_{n3}(y-y_0)]/\mu_{n3}]H(y-y_0)\}. \end{aligned} \tag{29}$$

Proceeding as in the first sub-problem, setting $P = 0$, we obtain the analytical solutions of sub-problems 2 and 3 as

$$\begin{aligned} w_2(x,y) = \frac{1}{2DR}\sum_{n=1}^{\infty}\sin(\alpha_n x) \times \Bigg\{&E_n\left[\frac{e^{\mu_{n3}(2b-y)} - e^{\mu_{n3}y}}{e^{2b\mu_{n3}} - 1} - \frac{e^{\mu_{n1}(2b-y)} - e^{\mu_{n1}y}}{e^{2b\mu_{n1}} - 1}\right] \\ &+ F_n[\operatorname{csch}(b\mu_{n3})\sinh(\mu_{n3}y) - \operatorname{csch}(b\mu_{n1})\sinh(\mu_{n1}y)]\Bigg\} \end{aligned} \tag{30}$$

and

$$\begin{aligned} w_3(x,y) = \frac{1}{2DR}\sum_{n=1}^{\infty}\sin(\beta_n y) \times \Bigg\{&G_n\left[\frac{e^{\mu'_{n3}(2a-x)} - e^{\mu'_{n3}x}}{e^{2a\mu'_{n3}} - 1} - \frac{e^{\mu'_{n1}(2a-x)} - e^{\mu'_{n1}x}}{e^{2a\mu'_{n1}} - 1}\right] \\ &+ H_n[\operatorname{csch}(b\mu'_{n3})\sinh(\mu'_{n3}y) - \operatorname{csch}(b\mu'_{n1})\sinh(\mu'_{n1}y)]\Bigg\} \end{aligned} \tag{31}$$

where $\beta_n = n\pi/b$, $\mu'_{n1} = \sqrt{\beta_n^2 + R}$, $\mu'_{n3} = \sqrt{\beta_n^2 - R}$.

3.2. Derivation of the analytical solutions for the original problem

To satisfy the boundary conditions at the four clamped edges, the sum of the slopes of the three sub-problems must vanish at each edge, which yields four infinite systems of simultaneous equations.

For the edge $y = 0$, we have, for $i = 1, 2, 3, \ldots$,

$$\begin{aligned} &\frac{P\sin(\alpha_i x_0)}{aDR}\{\sinh[\mu_{i3}(b-y_0)]\operatorname{csch}(b\mu_{i3}) - \sinh[\mu_{i1}(b-y_0)]\operatorname{csch}(b\mu_{i1})\} \\ &\quad + \frac{1}{2DR}\{E_i[\mu_{i1}\coth(b\mu_{i1}) - \mu_{i3}\coth(b\mu_{i3})] + F_i[\mu_{i3}\operatorname{csch}(b\mu_{i3}) - \mu_{i1}\operatorname{csch}(b\mu_{i1})]\} \\ &\quad + \frac{2\alpha_i}{aD}\sum_{n=1}^{\infty}\{\beta_n[G_n - \cos(i\pi)H_n]/[(\alpha_i^2 + \beta_n^2)^2 - R^2]\} = 0 \end{aligned} \tag{32}$$

where $\alpha_i = i\pi/a$, $\mu_{i1} = \sqrt{\alpha_i^2 + R}$, $\mu_{i3} = \sqrt{\alpha_i^2 - R}$.

For the edge $y = b$, we have, for $i = 1, 2, 3, \ldots$,

$$\frac{P\sin(\alpha_i x_0)}{aDR}\{\sinh(\mu_{i1}y_0)\mathrm{csch}(b\mu_{i1}) - \sinh(\mu_{i3}y_0)\mathrm{csch}(b\mu_{i3})\}$$
$$+ \frac{1}{2DR}\{E_i[\mu_{i1}\mathrm{csch}(b\mu_{i1}) - \mu_{i3}\mathrm{csch}(b\mu_{i3})] + F_i[\mu_{i3}\coth(b\mu_{i3}) - \mu_{i1}\coth(b\mu_{i1})]\}$$
$$+ \frac{2\alpha_i}{aD}\sum_{n=1}^{\infty}\{\beta_n\cos(n\pi)[G_n - \cos(i\pi)H_n]/[(\alpha_i^2 + \beta_n^2)^2 - R^2]\} = 0. \quad (33)$$

For the edge $x = 0$, we have, for $j = 1, 2, 3, \ldots$,

$$\frac{P\sin(\beta_j y_0)}{bDR}\{\sinh[\mu'_{j3}(a - x_0)]\mathrm{csch}(a\mu'_{j3}) - \sinh[\mu'_{j1}(a - x_0)]\mathrm{csch}(a\mu'_{j1})\}$$
$$+ \frac{1}{2DR}\{G_j[\mu'_{j1}\coth(a\mu'_{j1}) - \mu'_{j3}\coth(a\mu'_{j3})] + H_j[\mu'_{j3}\mathrm{csch}(a\mu'_{j3}) - \mu'_{j1}\mathrm{csch}(a\mu'_{j1})]\}$$
$$+ \frac{2\beta_j}{bD}\sum_{m=1}^{\infty}\{\alpha_m[E_m - \cos(j\pi)F_m]/[(\alpha_m^2 + \beta_j^2)^2 - R^2]\} = 0 \quad (34)$$

where $\alpha_m = m\pi/a$, $\beta_j = j\pi/b$, $\mu'_{j1} = \sqrt{\beta_j^2 + R}$, $\mu'_{j3} = \sqrt{\beta_j^2 - R}$.

For the edge $x = a$, we have, for $j = 1, 2, 3, \ldots$,

$$\frac{P\sin(\beta_j y_0)}{bDR}\{\sinh(\mu'_{j1}x_0)\mathrm{csch}(a\mu'_{j1}) - \sinh(\mu'_{j3}x_0)\mathrm{csch}(a\mu'_{j3})\}$$
$$+ \frac{1}{2DR}\{G_j[\mu'_{j1}\mathrm{csch}(a\mu'_{j1}) - \mu'_{j3}\mathrm{csch}(a\mu'_{j3})] + H_j[\mu'_{j3}\coth(a\mu'_{j3}) - \mu'_{j1}\coth(a\mu'_{j1})]\}$$
$$+ \frac{2\beta_j}{bD}\sum_{m=1}^{\infty}\{\alpha_m\cos(m\pi)[E_m - \cos(j\pi)F_m]/[(\alpha_m^2 + \beta_j^2)^2 - R^2]\} = 0. \quad (35)$$

From the above equations, E_m, F_m, G_n and H_n $(m, n = 1, 2, 3, \ldots)$ are obtained, substitution of which into Eqs. (29)–(31) gives the analytical solution of the original problem by

$$w(x, y) = w_1(x, y) + w_2(x, y) + w_3(x, y). \quad (36)$$

3.3. Numerical example

To verify the accuracy of the developed method, a square thin plate with $\nu = 0.3$ resting on an elastic foundation with $Ka^4/D = 10^2$ subjected to a concentrated load P at $(a/2, a/2)$ is examined. The results are tabulated in Tables 1–2 for comparison with those from the finite element method (FEM) by the ABAQUS package. The shell element S4R5 that imposes the Kirchhoff constraint numerically is used and 160 000 elements are adopted to obtain the convergent solutions as accurate as possible.

Convergence study shows that satisfactory accuracy is obtained when n is taken up to 20 for the deflections while taken up to 30 for the bending moments (m is taken to equal n for convenience). It should be pointed out that the moment at $(a/2, a/2)$, i.e., the concentrated loading position, in Table 2 does not converge just as it does in a simply supported plate [1]. It is clear that the solutions from the symplectic superposition method agree quite well with those from the FEM, which confirms the validity and accuracy of the present method.

4. Conclusions

The symplectic superposition method is developed in this paper to obtain the analytical bending solutions of clamped rectangular thin plates resting on elastic foundations. The solution approach reveals several advantages in handling the engineering problems such as the bending of rectangular plates on elastic foundations. Firstly, the symplectic superposition method provides a totally rational way to analytical solutions, which starts from the basic elasticity equations of the problem and proceeds without any preset solutions. The second advantage is that the method gives us a systematic solution procedure, which can be applied to other plate bending problems with classical edge support conditions. In addition, the method is expected to be extended to vibration and buckling problems, which will be developed in the follow-up work.

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Table 1
Deflections $w(Pa^2/D)$ for a clamped square thin plate resting on an elastic foundation.

y		x	$a/8$	$a/4$	$3a/8$	$a/2$
$a/8$	Present	$n = 10$	0.0000964	0.000344	0.000597	0.000706
		$n = 20$	0.0000964	0.000344	0.000597	0.000706
		$n = 30$	0.0000964	0.000344	0.000597	0.000706
	FEM		0.0000964	0.000344	0.000597	0.000706
$a/4$	Present	$n = 10$	0.000344	0.00112	0.00192	0.00228
		$n = 20$	0.000344	0.00112	0.00192	0.00228
		$n = 30$	0.000344	0.00112	0.00192	0.00228
	FEM		0.000344	0.00112	0.00192	0.00228
$3a/8$	Present	$n = 10$	0.000597	0.00193	0.00338	0.00410
		$n = 20$	0.000597	0.00192	0.00338	0.00410
		$n = 30$	0.000597	0.00192	0.00338	0.00410
	FEM		0.000597	0.00192	0.00338	0.00410
$a/2$	Present	$n = 10$	0.000699	0.00229	0.00410	0.00523
		$n = 20$	0.000706	0.00228	0.00410	0.00527
		$n = 30$	0.000706	0.00228	0.00410	0.00527
	FEM		0.000706	0.00228	0.00410	0.00527

Table 2
Bending moments $M_y(P)$ for a clamped square thin plate resting on an elastic foundation.

y		x	0	$a/8$	$a/4$	$3a/8$	$a/2$
0	Present	$n = 10$	0	−0.00937	−0.0519	−0.0960	−0.115
		$n = 20$	0	−0.00963	−0.0519	−0.0959	−0.115
		$n = 30$	0	−0.00961	−0.0518	−0.0959	−0.115
	FEM		0	−0.0100	−0.0516	−0.0952	−0.114
$a/8$	Present	$n = 10$	−0.00281	−0.0134	−0.0281	−0.0427	−0.0492
		$n = 20$	−0.00289	−0.0134	−0.0281	−0.0427	−0.0492
		$n = 30$	−0.00290	−0.0134	−0.0281	−0.0427	−0.0492
	FEM		−0.00295	−0.0134	−0.0281	−0.0427	−0.0492
$a/4$	Present	$n = 10$	−0.0156	−0.00880	−0.00271	−0.000677	−0.00225
		$n = 20$	−0.0156	−0.00881	−0.00270	−0.000674	−0.00226
		$n = 30$	−0.0156	−0.00881	−0.00270	−0.000674	−0.00226
	FEM		−0.0155	−0.00881	−0.00269	−0.000674	−0.00226
$3a/8$	Present	$n = 10$	−0.0288	−0.00379	0.0263	0.0572	0.0658
		$n = 20$	−0.0288	−0.00397	0.0266	0.0572	0.0651
		$n = 30$	−0.0288	−0.00397	0.0266	0.0572	0.0651
	FEM		−0.0286	−0.00397	0.0266	0.0572	0.0651
$a/2$	Present	$n = 10$	−0.0344	0.00128	0.0441	0.116	–
		$n = 20$	−0.0344	0.00129	0.0441	0.118	–
		$n = 30$	−0.0344	0.00129	0.0441	0.118	–
	FEM		−0.0342	0.00128	0.0441	0.118	–

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